EDF TESTS OF FIT FOR THE LOGISTIC DISTRIBUTION

BY

MICHAEL A. STEPHENS

TECHNICAL REPORT NO. 36

SEPTEMBER 5, 1979

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SUMMARY

In this paper we present goodness of fit tests for the logistic distribution, based on statistics calculations from the empirical distribution function. Emphasis is on the statistics W^2, U^2 and A^2, for which asymptotic percentage points are given, for each of the three cases where one or both of the parameters of the distribution must be estimated from the data. Slight modifications of the calculated statistics are given to enable the points to be used with small samples. Monte Carlo results are included also for statistics D^+, D^-, D and V .

Some key words: Cramér-von Mises statistics; Empirical distribution function; goodness-of-fit; Kolmogorov-Smirov Statistics; logistic distribution.

INTRODUCTION

In this paper we discuss the test of fit of H_0: that a random sample of n values of x comes from the logistic distribution

$$F(x) = 1/[1 + \exp\{-(x-\alpha)/\beta\}] \ , \quad -\infty < x < \infty \quad , \tag{1}$$

with one or both of the parameters α and β possibly unknown.

The tests given are based on statistics which measure the discrepancy between the theoretical distribution function (1), with estimates inserted for any unknown parameters, and the empirical distribution function (EDF) of the sample. Emphasis is placed on the statistics W^2, U^2 and A^2, for which asymptotic theory is derived; the theory is supported by Monte Carlo results to give percentage points for finite n. These tests are described in Sections 2 to 6. In Section 7, some Monte Carlo results are given for the Kolmogorov and Kuiper statistics D^+, D^-, D and V, for no which no asymptotic theory is available.

In deriving the asymptotic distributions of the statistics W^2, U^2 and A^2, the asymptotic theory of the empirical process is used. This is now very well known (see e.g. Durbin, 1973) and only the details applicable to the logistic distribution (1) will be repeated. The plan of the paper closely follows that of Stephens (1976), which was concerned with tests for normality and exponentiality, and in which much greater detail is given of the steps in the calculations.

2. THE GOODNESS OF FIT TESTS.

The null hypothesis H_0 is that the sample of x-values is a random sample from the logistic distribution (1). It will be convenient to suppose the sample is then labelled in ascending order, i.e., $x_1 < x_2 < \cdots < x_n$.

Four test situations may be distinguished, parallel to those in Stephens (1974, 1976, 1977): case 0, where both α and β are known, so that $F(x)$ is completely specified; case 1, where β is known and α is to be estimated; case 2, where α is known and β is to be estimated; case 3, where both α and β are unknown, and must be estimated.

Suppose that the parameters are estimated from the sample by maximum likelihood; the estimates, for Case 3, are given by the equations

$$n^{-1} \Sigma_i [1 + \exp\{(x_i - \hat{\alpha})/\hat{\beta}\}]^{-1} = 0.5 \tag{2}$$

$$n^{-1} \Sigma_i \left(\frac{x_i - \hat{\alpha}}{\hat{\beta}}\right) \frac{1 - \exp\{(x_i - \hat{\alpha})/\hat{\beta}\}}{1 + \exp\{(x_i - \hat{\alpha})/\hat{\beta}\}} = -1 \tag{3}$$

These two equations may be solved iteratively; suitable starting values for $\hat{\alpha}$ and $\hat{\beta}$ are $\alpha_0 = \bar{x}$ and $\beta_0 = s^2$, where $\bar{x}$ and s^2 are respectively the sample mean and variance. In case 1, β is known, and (2) is used for $\hat{\alpha}$ with β replacing $\hat{\beta}$. In Case 2, α is known; $\hat{\beta}$ is given by solving (3) with α replacing $\hat{\alpha}$.

When the parameters have been estimated as necessary, the steps in testing H_0 are as follows:

(a) Calculate $z_i = F(x_i)$, where $F(x)$ is given in (1), with the appropriate estimates inserted for unknown parameters in cases 1, 2 or 3. The z_i will now be in ascending order. Let $\bar{z}$ be the mean of the z_i.

(b) Calculate the test statistic desired:

$$W^2 = \Sigma_i \left(z_i - \frac{2i-1}{2n}\right)^2 + \frac{1}{12n} \, ; \; U^2 = W^2 - n\left(\bar{z} - \frac{1}{2}\right)^2 ;$$

$$A^2 = - \, [\Sigma_i (2i-1)\{\log z_i + \log(1 - z_{n+1-i})\}]/n - n \, .$$

(c) Refer to Table 1, first calculating the modified statistic and then comparing the result with the upper tail points given in the table, for the appropriate case. For example, suppose that with a sample of size 20, Case 2, the value of A^2 is 2.150; the modification involves calculating $(0.6)(20)(2.150)-1.8$ for the numerator, giving the value 24.0 and $(0.6)(20)-1.0$ for the denominator, giving value 11.0; the resulting modified A^2 is 2.182, which would be significant at about the 6% level. The modifications make only slight changes to the given value of a statistic, but they make it possible to dispense with tables of points for each n.

3. ASYMPTOTIC THEORY OF THE TESTS.

We first discuss the asymptotic theory of the empirical process; this is defined as $y_n(z) = \sqrt{n}(F_n(z) - z)$ where $F_n(z)$ is the EDF of the set z_i. Asymptotically $y_n(z)$ tends to a Gaussian process $y(z)$, defined for $0 \leq z \leq 1$; the mean of this process is zero, and values $y(0)$ and $y(1)$ are fixed, equal to zero; the covariance function of the process depends on the distribution tested and the parameters estimated. When the parameters are location and scale parameters, as are α and β for distribution (1), and when these are estimated by asymptotically efficient methods, the covariances are straightforward to find, and are independent of the true values of the parameters. The type of calculation is shown in detail in Stephens (1976, 1977).

First the functions $g_1(s) = \delta s/\delta\alpha$ and $g_2(s) = \delta s/\delta\beta$ are required, where $s = F(x)$. Since the final covariances do not depend on the parameters, α and β will be assumed to be 0 and 1 respectively. For distribution (1) the functions are

$$g_1(s) = s(s-1); \quad g_2(s) = s(s-1) \ln\{(1-s)/s\}.$$

The covariances of the asymptotic empirical process are then found to be

$$\begin{aligned}
&\text{Case 0:} \quad \rho_0(s,t) = s - st \\
&\text{Case 1:} \quad \rho_1(s,t) = \rho_0(s,t) - \phi_1(s)\phi_1(t) \\
&\text{Case 2:} \quad \rho_2(s,t) = \rho_0(s,t) - \phi_2(s)\phi_2(t) \qquad\qquad 0 \le s \le t \le 1 \\
&\text{Case 3:} \quad \rho_3(s,t) = \rho_0(s,t) - \phi_1(s)\phi_1(t) - \phi_2(s)\phi_2(t) \quad ,
\end{aligned}$$

where $\phi_1(s) = \sqrt{3}\, g_1(s)$ and $\phi_2(s) = \{3/(3+\pi^2)\}^{1/2} g_2(s)$. The covariance for case 3 takes a relatively simple form because the estimates $\hat{\alpha}$ and $\hat{\beta}$ are asymptotically uncorrelated.

The asymptotic distribution of W^2, for any of the cases 0,1,2 or 3, may then be expressed as

$$W^2 = \sum_{i=1}^{\infty} \frac{u_i^2}{\lambda_i} \tag{4}$$

where u_i are independent standard normal variables, and where for case j, the weights λ_i are the eigenvalues of the integral equation

$$f(s) = \lambda \int_0^1 \rho_j(s,t)\, f(t)\, dt \, . \tag{5}$$

Asymptotic points for W^2 are therefore found by solving (5) for the weights λ_i, and then calculating the percentage points of the distribution (4).

4. CALCULATION OF THE WEIGHTS.

The weights λ_i are found as follows. Let $0 < \lambda_1 < \lambda_2 < \ldots$ be the weights and $f_1(x), f_2(x), \ldots$ be the associated normalized eigenfunctions for case 0. For W^2, these are $\lambda_i = \pi^2 i^2$ and $f_i(x) = \sqrt{2}\sin(\pi i x)$. Set

$$D_0(\lambda) = \Pi_i(\lambda-\lambda_i) .$$

Expand $\phi_1(x) = \Sigma_i a_i f_i(x)$ and $\phi_1(x) = \Sigma_i b_i f_i(x)$, so that

$$a_i = \int_{-\infty}^{\infty} \phi_1(x) f_i(x)\,dx, \quad b_i = \int_{-\infty}^{\infty} \phi_2(x) f_i(x)\,dx.$$

Let

$$S_a(\lambda) = 1 + \lambda\Sigma_i \frac{a_i^2}{\lambda-\lambda_i}, \text{ and } S_b(\lambda) = 1 + \lambda\Sigma_i \frac{b_i^2}{\lambda-\lambda_i} .$$

In general, the λ_i of cases 1 and 2 are then given by setting the Fredholm determinant to zero; this implies, for case 1, solving $D_0(\lambda)S_a(\lambda) = 0$, and for case 2, $D_0(\lambda)S_b(\lambda) = 0$. For the distribution (i), $\phi_1(s) = \phi_1(1-s)$ and $\phi_2(s) = -\phi_2(1-s)$; it follows that $a_i = 0$ for i an even integer, and $b_i = 0$ for i an odd integer; thus $a_i b_i = 0$ for all i. When this occurs the weights for case 3 are solutions of $T(\lambda) = 0$, where $T(\lambda) = D_0(\lambda)S_a(\lambda)S_b(\lambda)$.

For W^2 case 1, because $a_i = 0$, the value $\lambda = \lambda_i$, for i even, is a solution of $D_0(\lambda)S_a(\lambda) = 0$; but for i odd, the term $\lambda - \lambda_i$ in $D_0(\lambda)$ cancels the denominator in the term in $S_a(\lambda)$ containing a_i, and the product $D_0(\lambda_i)S_a(\lambda_i)$ is not zero. Thus solutions for case 1 contain the set λ_i, i even, and another set of λ values which are the solutions of $S_a(\lambda) = 0$. Similarly for case 2, solutions of $D_0(\lambda)S_b(\lambda) = 0$ consist of two sets: $\lambda = \lambda_i$, i odd, and another set, the solution of $S_b(\lambda) = 0$.

It follows that for case 3 the solutions of $T(\lambda) = 0$ do not contain any of the standards λ_i, but only the values λ satisfying $S_a(\lambda) = 0$ and $S_b(\lambda) = 0$ which have already been found for cases 1 and 2.

The statistic A^2 is a functional of a process closely related to the empirical process, for which the asymptotic covariances are the same as those for W^2, multiplied by $w(s,t) = \{(s-s^2)(t-t^2)\}^{-1/2}$. The asymptotic distribution is then of type (4), with the weights λ_i calculated from (5) with the new covariances. The functions $f_i(x)$ for case 0 are $P_i^!(2x-1)$ where $P_i^!(t)$ are Ferrar associated Legendre functions, and the standard λ_i is $i(i+1)$. Functions $\phi_i(x)$ and $\phi_2(x)$ are expanded in terms of these $f_i(x)$ to obtain the coefficients a_i and b_i, as for W^2. Solutions for A^2, case 1 are then the two sets given by λ_i, i even and the solutions of $S_a(\lambda) = 0$; for case 2 they are λ_i, i odd, and the solutions of $S_b(\lambda) = 0$; and for case 3 they are the solutions of $S_a(\lambda) = 0$ and of $S_b(\lambda) = 0$.

For U^2, the discussion is more complicated. The statistic depends on the asymptotic process $y(z) - \int_0^1 y(u)du$. The covariances of this process for the various cases are then

Case 0: $\rho_0(s,t) + a(s,t)$

Case 1: $\rho_0(s,t) + a(s,t) = (\phi_1(s) + \sqrt{3}/6)(\phi_1(t) + \sqrt{3}/6)$

Case 2: $\rho_0(s,t) + a(s,t) - \phi_2(s)\phi_2(t)$

Case 3: $\rho_0(s,t) + a(s,t) - (\phi_1(s) + \sqrt{3}/6)(\phi_1(t) + \sqrt{3}/6) - \phi_2(s)\phi_2(t)$

where $a(s,t) = 1/12 + \{s(s-1) + t(t-1)\}/2$. The asymptotic distribution of U^2 for a particular case is then of the type in equation (4) with λ_i calculated from (5) using the appropriate covariance.

For case 0, the standard roots λ_i are double roots with values $\lambda_i = 4\pi^2 i^2$, and the corresponding eigenfunctions are $f_i(x) = \sqrt{2}\sin(2\pi ix)$ and $f_i^*(x) = \sqrt{2}\cos(2\pi ix)$. Then define

$$a_i = \int_{-\infty}^{\infty} \phi_1(x)\ \sqrt{2}\ \sin(2\pi ix)\ dx,\quad a_i^* = \int_{-\infty}^{\infty} \phi_1(x)\ \sqrt{2}\ \cos(2\pi ix)\ dx$$

and

$$S_a(\lambda) = 1 + \lambda\ \Sigma_i \frac{a_i^2}{\lambda-\lambda_i} + \lambda\ \Sigma_i \frac{a_i^{*2}}{\lambda-\lambda_i} \quad ,$$

with corresponding expressions, using $\phi_2(x)$, for b_i, b_i^* and $S_b(\lambda)$. An analysis similar to that for W^2 now shows that standard roots λ_i, for all i, will occur once each in the solutions of $D_0(\lambda)S_a(\lambda) = 0$ for case 1; another set is given by the solutions of $S_a(\lambda) = 0$. Similarly standard roots occur once in case 2, and another set is given by $S_b(\lambda) = 0$. For case 3, for which again $D_0(\lambda)S_a(\lambda)S_b(\lambda) = 0$, only the set given by $S_a(\lambda) = 0$ and by $S_b(\lambda) = 0$ will be solutions.

However, there is an interesting special result which for the logistic distribution (1) makes it unnecessary to perform some of the above calculations for U^2. This follows because equation (2) may be written $\Sigma_i F(x_i) = n/2$; thus in the notation of step (a), Section 2, we have $\bar{z} = 0.5$, and so $W^2 = U^2$. Equation (2) is satisfied by $\hat{\alpha}$ and β in case 1, and by $\hat{\alpha}$ and $\hat{\beta}$ in case 3, so for these cases $W^2 = U^2$ for all n, and therefore they have the same distributions. The asymptotic covariances for W^2 and U^2 become identical, as they should, when $\phi_1(s) = \sqrt{3}\, s(1-s)$ is inserted in these expressions. Thus the theory for U^2 needs to be worked out independently only for case 2.

5. CALCULATIONS OF EXACT MEANS AND VARIANCES.

The cumulants K_j of the distribution (4) may be expressed in two ways: the first is

$$K_j = 2^{j-1}(j-1)! \int_0^1 \rho_j(s,s)ds,$$

where $\rho_1(s,t) = \rho(s,t)$, and, for $j \geq 2$, $\rho_j(s,t)$ is defined by

$$\rho_j(s,t) = \int_0^1 \rho_{j-1}(s,u)\ \rho(u,t)du .$$

A second form for K_j is $K_j = 2^{j-1}(j-1)!\ \Sigma_i (1/\lambda_i)^j$.

From the first of these formulas the means and variances of the asymptotic distributions may be found, although the algebra, especially for the variances, is sometimes very tedious. For example, for W, the mean μ_j for case j is $\mu_0 - \Delta_j$, where Δ_j is

$$\Delta_j = \int_0^1 \phi_j^2(s)ds, \qquad (j = 1,2) ,$$

and

$$\Delta_3 = \Delta_1 + \Delta_2 .$$

Evaluation of the Δ integrals gives for the four means of W^2 :

$\mu_0 = \frac{1}{6}$, $\mu_1 = 0.06667$, $\mu_2 = 0.14825$, $\mu_3 = 0.048253$. For U^2, similar analysis gives $\mu_0 = \frac{1}{12}$, $\mu_1 = 0.06667$, $\mu_2 = 0.06492$, $\mu_3 = 0.048253$. The means of W^2 and U^2 are the same for cases 1 and 3, as they should be from the identity of the distributions noted above.

For A^2, corresponding calculations give the means $\mu_0 = 1$, $\mu_1 = 0.5$, $\mu_2 = 0.84966$, $\mu_3 = 0.34966$.

Note that the series formula for K_j can be used, once the weights λ_j are known, to find the cumulants to any order, whereas the integration needed for the first formula for K_j rapidly becomes prohibitive. However the series for the mean K_1 converges very slowly, and it is worthwhile to calculate the exact means as was done above. For higher values of j the series converges more quickly and the weights may be used to find the variances and higher cumulants. In some cases, the variances were also calculated using the integrals, as a check on the calculations using the weights.

6. CALCULATION OF PERCENTAGE POINTS.

When the λ_i and the exact means are known for one of the asymptotic distributions the percentage points of the distribution (4) can be found by several approximating techniques. A common method for finding the distribution of the sum of weighted χ^2 variables is Imhof's (1961) method, which has been adapted by Durbin & Knott (1972) for the case of an infinite sum. Alternatively the first four cumulants can be found and Pearson curves fitted to the data. In terms of α-values the points given by this method differ negligibly from those given by Imhof's method, and the latter are those quoted in Table 1. Points for case 0 are included to show how much the percentage points drop in the other cases; they demonstrate how important it is to use the correct values for the appropriate case.

For the percentage points of the various statistics for finite n, Monte Carlo studies were made, with $n = 5, 8, 10, 20$ and 50; 10,000 samples were used for each case. Previous experience with these statistics (Stephens, 1974, 1977) suggests that convergence to the asymptotic points will be rapid, and a plot of percentage points against $1/n$ proves this to be so. The Monte Carlo points were used to calculate the modified forms given in Table 1; it can be seen that several models were employed to connect the percentage point of a test statistic for sample size n with its asymptotic point. It is believed that use of these modifications and the appropriate asymptotic point will give an error in α always less than 0.5%, for the upper tail given, and for $n > 5$.

7. KOLMOGOROV-SMIRNOV STATISTICS

Another important group of EDF statistics contains those associated with the names of Kolmogorov, Smirnov and Kuiper; these are statistics D^+, D^-, D and V defined in terms of the z_i of section 2 as follows:

$$D^+ = \max_i (i/n - z_i); \qquad D^- = \max_i \{z_i - \frac{(i-1)}{n}\};$$

$$D = \max(D^+, D^-); \qquad V = D^+ + D^- .$$

The first three of these are older than statistics W^2, U^2 and A^2, and have attracted more attention. Statistic V was introduced by Kuiper for observations on a circle, since the value of V, like that of U^2, is independent of the choice of origin; both statistics can also be used for observations on a line. Although asymptotic distribution theory cannot be offered for these statistics for the cases 1, 2, and 3 of this paper, Monte Carlo results were obtained at the same time as for W^2, U^2 and A^2. The percentage points of $\sqrt{n}D^+$, $\sqrt{n}D$ and $\sqrt{n}V$ are given in Table 2. Points for $\sqrt{n}D^+$ can be used also for $\sqrt{n}D^-$. For each statistic and case, the Monte Carlo percentage point at level α was plotted against $1/n$ and smoothed to give the points in the table. The asymptotic values are found by extrapolation, and the accuracy is therefore somewhat difficult to determine. In other goodness-of-fit situations (see, e.g. Stephens, 1974) these statistics have been found to be less powerful than W^2, U^2 or A^2, and so they are not particularly recommended. However, the points are included because these statistics often have a pictorial appeal to practical users, and D in particular is very well known; also D^+ and D^- enable one-sided tests to be made. The values will also be of interest to statisticians working in this field, particularly if asymptotic results later become available.

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Table 1

Percentage points for modified statistics W^2, U^2, A^2.

Statistic	Case	Modification	0.75	0.90	0.95	0.975	0.99	.995
			Upper tail percentage points, α					
W^2	0	$(W^2-0.4/n+0.6/n^2)(1.0+1.0/n)$	0.209	0.347	0.461	0.581	0.743	0.869
	1	$(1.9nW^2-0.15)/(1.9n-1.0)$	.083	.119	.148	.177	.218	.249
	2	$(0.95nW^2-0.45)/(0.95n-1.0)$	.184	.323	.438	.558	.721	.847
	3	$(nW^2-0.08)/(n-1.0)$	.060	.081	.098	.114	.136	.152
U^2	0	$(U^2-0.1/n+0.1/n^2)(1.0+0.8/n)$	.105	0.152	0.187	0.221	0.267	.304
	2	$(1.6nU^2-0.16)/(1.6n-1.0)$	.080	.116	.145	.174	.214	.246
		For U^2 Cases 1 and 3 use modifications and percentage points for W^2 Cases 1 and 3 respectively (See Section 4).						
A^2	0	none	1.248	1.933	2.492	3.070	3.857	4.500
	1	$A^2+0.15/n$	.615	.857	1.046	1.241	1.505	1.710
	2	$(0.6nA^2-1.8)/(0.6n-1.0)$	1.043	1.725	2.290	2.880	3.685	4.308
	3	$A^2(1.0+0.25/n)$	.426	.563	.660	.769	.906	1.010

Table 2

Percentage points for statistics $\sqrt{n}\,D^+$, $\sqrt{n}\,D$, $\sqrt{n}\,V$,

Statistic	Case	Sample size	Upper tail percentage points: α-level .90	.95	.975	.99
$\sqrt{n}\,D^+$	1	5	.702	.758	.805	.854
		10	.730	.792	.846	.913
		20	.744	.809	.867	.944
		50	.752	.819	.880	.962
		∞	.757	.826	.888	.974
	2	5	.971	1.120	1.239	1.380
		10	.990	1.143	1.268	1.423
		20	.999	1.150	1.282	1.444
		50	1.005	1.161	1.290	1.456
		∞	1.009	1.166	1.297	1.464
	3	5	.603	.650	.690	.735
		10	.636	.687	.736	.789
		20	.653	.705	.758	.816
		50	.663	.716	.773	.832
		∞	.669	.723	.781	.842
$\sqrt{n}\,D$	1	5	.736	.791	.845	.883
		10	.777	.837	.895	.953
		20	.800	.865	.926	.997
		50	.808	.874	.937	1.011
		∞	.816	.883	.947	1.025
	2	5	1.108	1.236	1.349	1.474
		10	1.148	1.274	1.388	1.521
		20	1.167	1.294	1.406	1.545
		50	1.179	1.305	1.419	1.559
		∞	1.187	1.313	1.427	1.568

Table 2 (Continued)

Statistic	Case	Sample size	.90	.95	.975	.99
$\sqrt{n}\ D$	3	5	.643	.679	.723	.751
		10	.679	.730	.774	.823
		20	.698	.755	.800	.854
		50	.708	.770	.817	.873
		∞	.715	.780	.827	.886
$\sqrt{n}\ V$	1	5	1.369	1.471	1.580	1.658
		10	1.410	1.520	1.630	1.741
		20	1.433	1.550	1.659	1.790
		50	1.447	1.564	1.675	1.815
		∞	1.454	1.574	1.685	1.832
	2	5	1.314	1.432	1.547	1.674
		10	1.372	1.483	1.587	1.711
		20	1.400	1.510	1.607	1.730
		50	1.417	1.525	1.619	1.741
		∞	1.429	1.535	1.627	1.748
	3	5	1.170	1.246	1.299	1.373
		10	1.230	1.311	1.381	1.466
		20	1.260	1.344	1.422	1.514
		50	1.277	1.364	1.448	1.542
		∞	1.289	1.376	1.463	1.560

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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EDF TESTS OF FIT FOR THE LOGISTIC DISTRIBUTION

In this paper we present goodness of fit tests for the logistic distribution, based on statistics calculations from the empirical distribution function. Emphasis is on the statistics W^2, U^2 and A^2, for which asymptotic percentage points are given, for each of the three cases where one or both of the parameters of the distribution must be estimated from the data. Slight modifications of the calculated statistics are given to enable the points to be used with small samples. Monte Carlo results are included also for statistics D^+, D^-, D and V.

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